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3. **CULTIVATION OF CLIMATE THROUGH MATHEMATICS CLUBS**
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4. **SIGHTING THE VALUE OF π (using parallel lines to find π)**
   'The Mathematics Teacher', The National Council of Teachers of Mathematics, USA
   May 1981 Vol 74 No 5 pp 380-384

5. **DETECTION AND CARE OF THE GIFTED IN MATHEMATICS**
   The Mathematics, The National Council of Teachers of Mathematics, USA
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INNOVATIVE APPROACHES IN MATHEMATICS EDUCATION

Some Published Papers

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CRITERIA FOR INCLUSION OF AN ITEM IN MATHEMATICS CURRICULUM AT PRIMARY LEVEL

Introduction

It is common experience to encounter vested interest on the part of teachers, particularly at primary level, in perpetuating the type of school mathematics with which they were familiarised, even though it is not conducive to the developmental needs of children as viewed from their future.

Of course, the inevitability of changes in the primary mathematics curriculum is conceded, what with the phenomenal increase in mathematics users and the attitudinal changes witnessed in the community, professional and lay, on the advent of the computer. But the crucial issue is to accept changes which will naturally bring about deletion and inclusion, pushing down and pushing up of topics in the school curriculum.

What is in vogue today, by and large, is to give consideration, while framing the curriculum, to the nature of the society, the nature of the children, the nature of its teachers and the nature of mathematics. It is felt that there should be no reference to the methods of teaching in mathematics curriculum making. This may be acceptable in other subjects of the School Curriculum but when it concerns the mathematics component, strict adherence to omission of methods can be seen to vitiate the learning process that has to be engineered by the teachers. So it is of paramount importance to raise the question whether the four factors suffice. It has to be admitted, of course, that there is yet to emerge a curriculum where the four factors have been imaginatively balanced.

Communication gap

Teachers have since learnt to distinguish between a syllabus and a curriculum. In a curriculum, they look forward to listing of topics (or concepts) linked with behavioural outcomes on the part of children, mention

of learning - teaching aids with the associated process learning experiences and related evaluation that covers not only recall and repetition but also application in a new situation.

The expectations that were roused by the so called new maths programmes in the sixties did not materialise and the outcomes turned out to be disastrous. Why did that happen? It surfaced that the communication between the mathematicians and mathematics educators was not all that easy. Though two decades have passed, channels of communication have not yet been comfortably established as their concerns vary, if not conflict. This situation has fortunately seen the most welcome phenomenon of a few top ranking mathematicians coming forward to understand the problems of mathematics education and offering guidelines that would, without causing damage to the texture of mathematics that they cherish, help practising teachers to present mathematics in tune with the developmental make-up and shifting interests of children. For instance, the multi-embodimental principle of Skemp, problem solving approach of Polya, discovery-oriented learning of Bruner, non-formalism of Kinchin, life situation stance of Sawyer etc. have been contributing a lot in making teachers' task in establishing rapport with learners far easier.

Lack of emphasis

But it has to be observed that there has been no spelling out of criteria for inclusion of a topic in mathematics curriculum and the need is all the more greater at primary level. Only the backing of tradition or assertion of the professors lends weight in favour of giving a topic a place in the curriculum. The inappropriateness or the inadequacy presents itself when the text book writers or the classroom teachers handle the changes.

As one engaged in curriculum construction at State and National levels, I have often found myself explaining the various approaches to a topic and suitability or otherwise of some or one of them in introducing the

topic at the level being contemplated. Unfortunately, there exists, to my knowledge, no comprehensive reference material which deals with topics and various approaches to their presentation. If there could be international co-operation in this vital exercise, I venture to predict rapid developments in healthy and relevant curriculum-making in mathematics all over the globe to the great relief of all who are disturbed by the stagnating curricular efforts.

Some case studies

Ask any experienced teacher when one could introduce algebraic or symbolic language in school mathematics. He would say that it should be in the post primary stage as is obtaining now. How is it done? Is it not through equations or formulae? Butler, Wren and Banks affirm in their book, 'Teaching Mathematics in Secondary Schools' that 'there is much diversity of opinion as to whether algebra should start with formulae or equations'. But psychologists tell us that pattern finding is in the make-up of a child. So besides the approaches that involve <u>formulae</u> and <u>equations</u>, the approach of pattern language should also be considered. <u>Admissibility of a topic at any level often depends upon the appropriateness of approach to it.</u>

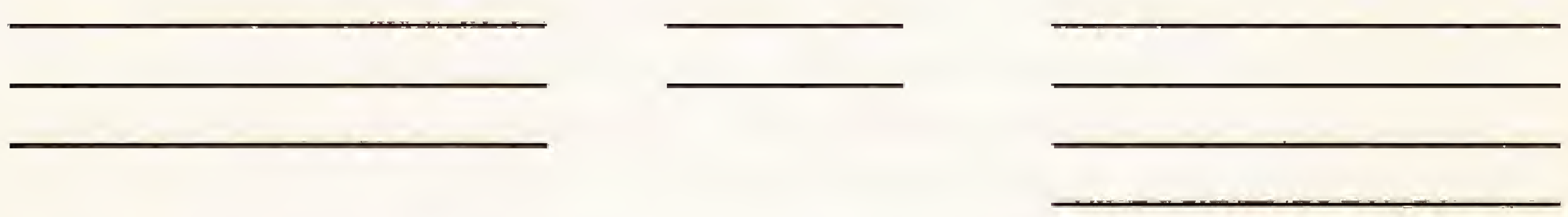
My experience tells me that through pattern language approach, algebraic expressions can be introduced without learner resistence at the third year itself when a child would have become sufficiently competent in handling whole numbers.

1. Ask children to examine which of the expressions $5 + 3$, $8 + 1$, $6 + 4 + 2$ etc. will go with $2 + 1$, $9+1$, $3 + 1$ etc. Children by and large are seen to plump for $8 + 1$ and the reason for their choice is also given. At this stage, the teacher introduces the pattern language $a + 1$, where <u>a</u> is a variable. This approach easily lends itself to presentation of expressions like $a + b$, $4a + 3$, $ab + bc + ca$, $a^2 + a + 1$ etc.

If this is followed by geometric designs also, the

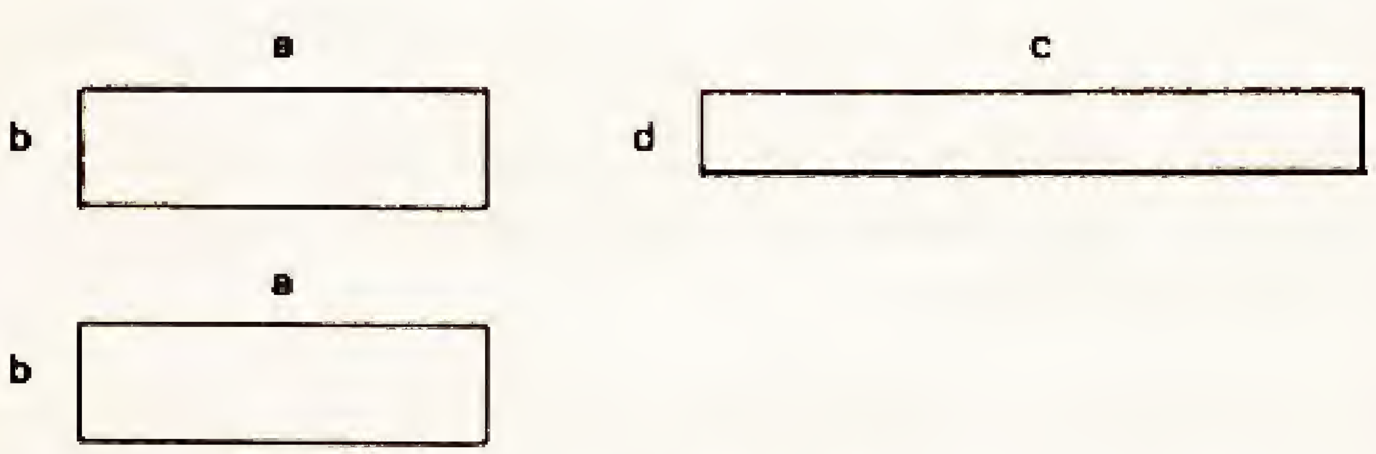
resourcefulness of algebraic expressions gets delightfully accepted and mastered.

For example, consider

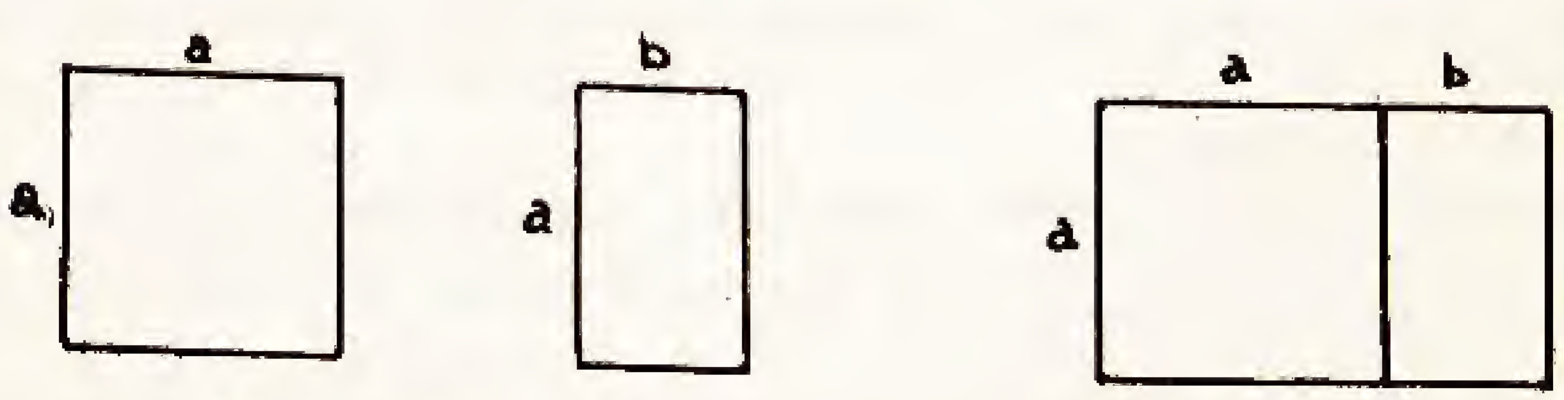


Children at once see the elegance in presenting the design as 3a + 2b + 4c, where letters represent lengths, different letters representing different lengths. The letter symbols get introduced as parametric variables. Instead of line segments, two dimensional figures like cuboids, cubes etc, can be used in setting up designs. By using one dimensional characterisation in two and three dimensional designs, expressions involving second and third degree terms can get introduced.

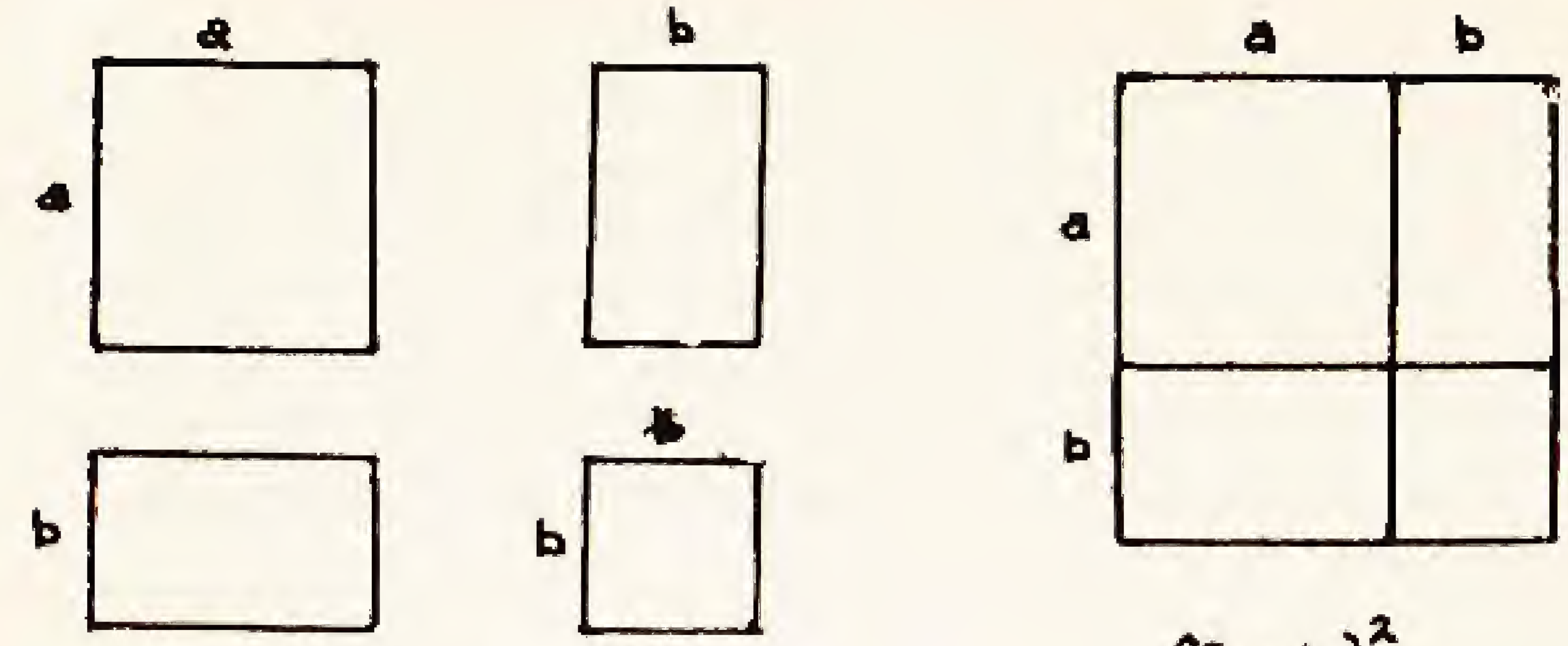
See the few typical illustrations given below:



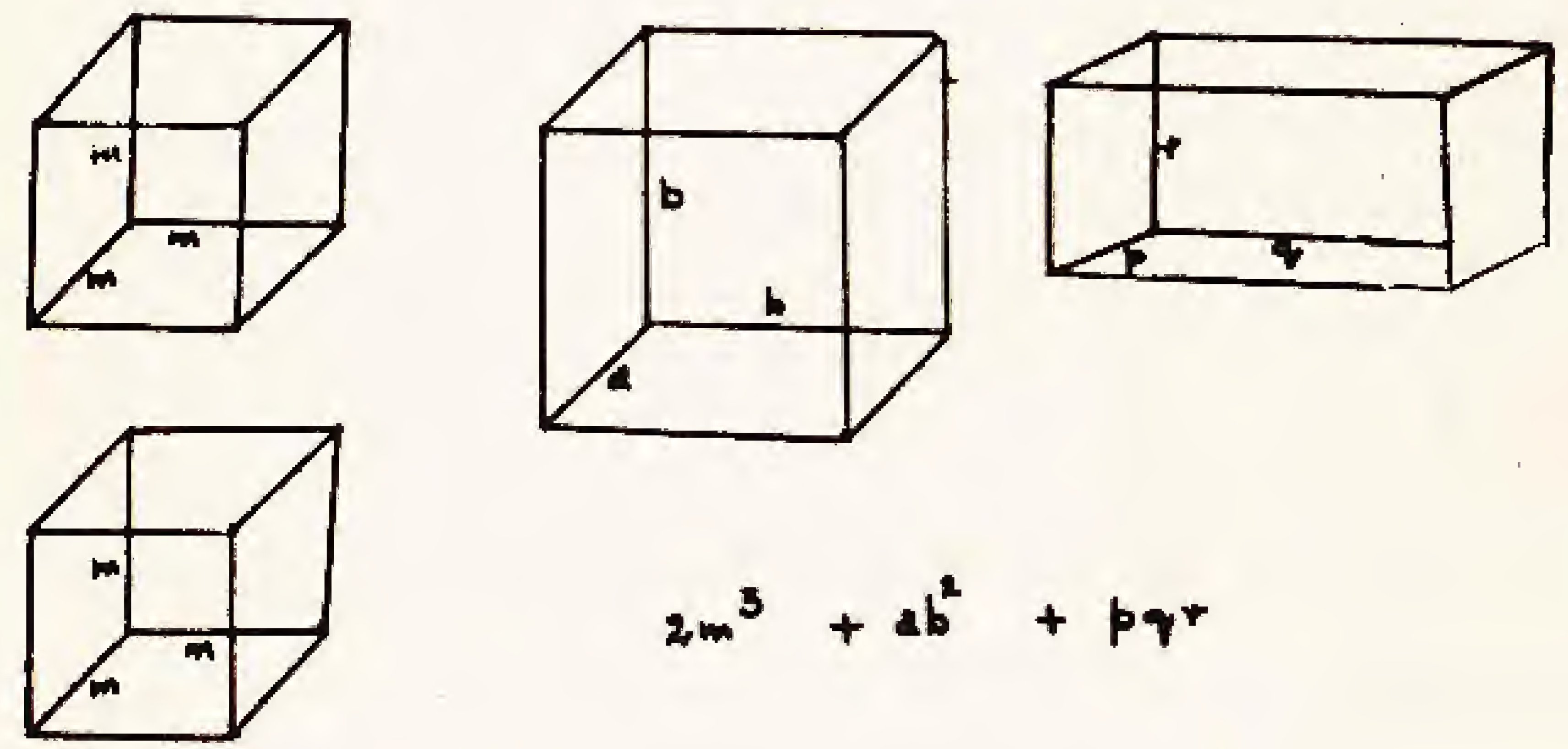
$2ab + cd$



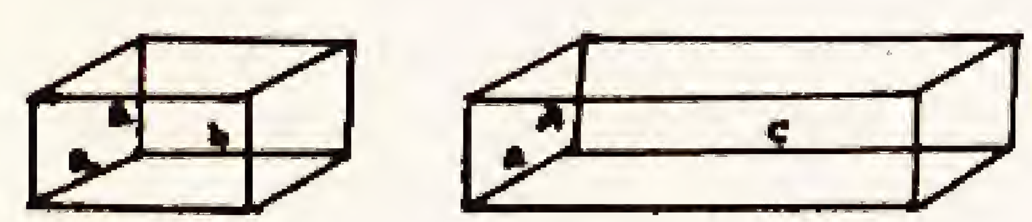
$a^2 + ab \qquad a(a+b)$



$a^2 + ab + ab + b^2 \qquad (a+b)^2$



$2m^3 + ab^2 + pqr$



$a^2b + a^2c$

$a^2(b+c)$

$a^2b + a^2c = a^2(b+c)$

2. When should children study logarithms? If an experienced teacher, why even a University Professor is asked this quesiton, it would be surprising to find one who would say that it could be done at class V level. Why? Again the question of approach decides the issue. It is good that indices are getting introduced at primary level in many countries. Study the tables given below (where products of repeated factors are given showing number of repeated factors or index).

Repeated factors	2	2x2	2x2x2	2x2x2x2	2x2x2x2x2	2x2x2x2x2x2
Products	2	4	8	16	32	64
No. of factors or index	1	2	3	4	5	6

Children are fascinated by the interesting discovery that lies within their competence.

$$\begin{array}{ccccc} 2 & \times & 4 & = & 8 \\ \downarrow & & \downarrow & & \uparrow \\ 1 & + & 2 & = & 3 \end{array}$$

This approach permits earlier introduction of logarithms.

Conclusion

The case studies clearly show that modes and levels of presentation of topics can alone to a large extent provide the justification for their inclusion. There should be enough choice in approaches for the curriculum framer as well as the teacher to tailor his teaching to the attainment and maturity of his pupils in the back drop of their growth and environment.

The other criteria should be (1) immediate and wide applicability (2) place in the main stream of mathematics (3) lending itself to concretisation and (4) suitability for independent learning.

References:

1 Butler, Wren and Banks (1970). *The Teaching of Secondary School Mathematics* - New York : Mc Graw Hill Book Co.

2 Johnson and Rising (1972) : *Guidelines for Teaching Mathematics,* California, Wadsworth Publishing Co.

THE PROBLEM OF ABRUPTNESS IN MATHEMATICS TEACHING

2

Psychological perspective

Learning in general and mathematical learning in particular has to be viewed as continual modification of knowledge structures. The whole process can be characterised as a scenario of transitions, thus becoming an avenue for management of transitions to maximise smooth learning, that is, learning with the least resistance. This entails avoidance of abruptness through deliberate and well planned provision of connecting links to facilitate information processing.

The development of mathematical thinking, though sequential stage-wise, is not linear but concentric, radiating streams of refinements. So if what is to emerge later can be indicated earlier, wherever possible and in whatever crude form it would have to be presented, the jolt of abruptness can be made to disappear to facilitate surfacing of connections and acceptance of apparently 'shocking' rules for the sake of preserving structures. Moreover, this is pedagogically sound, as it would be in keeping with the historical evolution of mathematical concepts.

The problem of abruptness in mathematical learning has not received enough attention, I believe, in the hands of curriculum planners, instruction designers and text book writers as a result of which the disconnected presentations have militated against integrated thinking and deprived learners of developing readiness for restructuring of acquired knowledge to develop new knowledge in a smooth transition.

A few areas in school mathematics are selected and dealt with in sufficent detail to defend this thesis.

Whole Numbers to Fractional Numbers

Many of the properties of fractional numbers and their rules of operation could be introduced while teaching whole numbers and their operations. Division in the set of whole numbers is indicated by $\div$ and if, besides this, the dividing bar notation of the fractional form is also used as an alternative to the symbol and if learners are encouraged to explore the patterns of addition and subtraction, multiplication and division of whole numbers with addends and factors expressed in division form using fractional notation, the learners can be seen developing readiness to map dividend, divisor and quotient onto numerator, denominator and fractional number and appreciate the consistency in the presentation of structural properties. Only in the case of remainder, the change in the system of fractional numbers would be dramatic, as, being zero always, it would cease to vary. Fractional number would then emerge as an entity by anticipation and not by abrupt introduction.

To start with, let the division indicated by **$6 \div 2$** have also an alternative form in transforming addition, subtraction, multiplication and division statements in the system of whole numbers. Examples are cited below by way of illustration.

Consider $3 + 5 = 8$ Rewriting 3 and 5 as divisions with the same divisor, say 2, we can write

$$3 + 5 = \frac{6}{2} + \frac{10}{2}$$

How to obtain 8, using 6, 10, 2 and 2 becomes a question for exploration. Students discover that by adding the dividends 6 and 10 only and dividing the sum by 2, 8 can be got.

$$3 + 5 = \frac{6}{2} + \frac{10}{2} = \frac{6 + 10}{2} = \frac{16}{2} = 8$$

The question that arises in this context is whether it would be possible to set 8 when 3 and 5 are expressed as divisions with different divisors as for instance 12/4 and 15/3. By trial and error exploration, it is discovered that unless the same divisor is used, the addition cannot be performed to get the required sum. Examining the subtraction situation, the experience is seen to be similar.

Now by writing,

$$3 = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4} = \frac{15}{5} = \frac{18}{6} = \frac{21}{7}$$

$$= \frac{24}{8} = \frac{27}{9} \text{ etc}$$

multiplication table of 3 in a different form, the principle that when the dividend and the divisor are multiplied by the same natural number or divided by their common factor, the same quotient is obtained surfaces and is spelled out. Later, this gets recognised as the property of equivalent fractions, once the concept of fractional number is presented.

In the case of multiplication, the constraint experienced in doing addition and subtraction is surprisingly found to be absent.

Consider next 5 x 8 = 40. Writing 5 and 8 as divisions with the same divisor, say 3, we can write

$$5 \times 8 = \frac{15}{3} \times \frac{24}{3}$$

There is a feeling of anticlimax as $\frac{15 \times 4}{3}$ does not yield the required product 40,

but only $\frac{15 \times 24}{3 \times 3}$ The feeling gets heightened when

the situation remains unchanged,
Even if the factors are rewritten as divisions with different divisors.

$$5 \times 8 = \frac{30}{6} \times \frac{16}{2} = \frac{30 \times 16}{6 \times 2} = \frac{480}{12} = 40$$

What remains now to be examined is just division. This operation surprisingly bristles with manipulative

complexity after a deceptive start when the dividend and divisor are expressed as divisions with the same divisor. Consider, 12 ÷ 3 = 4. Rewriting 12 and 3 as divisions, first with the same divisor, say 4 and then with different divisors, say 2 and 5, the oft-repeated thumb rule *invert and multiply* emerges after considerable trial and error.

$$12 - 3 = \frac{40}{4} - \frac{12}{4} = \frac{48 - 12}{4 - 4} = \frac{4}{1} = 4$$

(The rule does not emerge at this stage.)

$$12 - 3 = \frac{24}{2} - \frac{15}{5} = \frac{24 \times 5}{2 \times 15} = \frac{120}{30} = 4$$

$\frac{24 \times 5}{2 \times 15}$ is the same as $\frac{24}{2} \times \frac{5}{15}$ and $\frac{5}{15}$ is the inverted form of $\frac{15}{5}$ Examining if this holds good in the 'same divisor' case, we find $12\text{-}3 = \frac{48}{4} - \frac{12}{4} = \frac{48}{4} \times \frac{4}{12} = \frac{192}{48} = 4$,
showing that this pattern of manipulation covers both the cases and hence general.
Finally, in addition to the notation of writing
13 - 5 = 2R3, if 13-5=2 3/5 (read as 2 and 3 for 5) is also introduced, the transition from the system of whole numbers to the extended system of fractional numbers will be rendered smooth and meaningful. Once the notion of whole and part is introduced, 2 3/5 will take on a new meaning as 2 and 3 fifths, 2 3/5 itself becoming the quotient with null remainder. Allowing familiar things to assume new meanings for greater generality and power with some losses of course has been the underlying spirit of adventure in mathematical thinking.

Whole Numbers to Integers

Integers also lend themselves to this kind of anticipatory exercise. Any whole number can be expresses as a sum or difference of two whole numbers and that gives the initial start for managing the transition. a few examples are set out below:

Addition: 7 + 4 = 11

7 = 5 + 2	7 = 9 - 2
4 = 3 + 1	4 = 8 - 4
11 = 8 ...	11 = 17...
11 = 8 + 3	11 = 17 - 6
7 = 10 - 3	7 = 4 + 3
4 = 3 + 1	4 = 5 - 1
11 = 13	11 = 9 ....
11 = 13 - 2	11 = 9 + 2

7 and 4 are expressed as binary expressions involving addition or subtraction with two numbers. While adding, a sum is known and the first term of its binary expression is also easily obtained. The problem is to fix the second term and it is fixed with respect to the first term in order to obtain he required sum. A study is made to find out how the second terms in the two addends are computed to get the second term of the sum and this reveals a need to recognise the second terms with the + and - signs as entities and formulate their behaviour in addition. A familiar pattern seen in the addition of two integers emerges.

Subtraction: 9 - 4 = 5

	9 = 6 + 3	9 = 6 + 3
	4 = 2 + 2	4 = 5 - 1
	5 = 4 ...	5 = 1 ...
	5 = 4 + 1	5 = 1 + 4
9 = 13 - 4	9 = 13 - 4	9 = 15 - 6
4 = 6 - 2	4 = 3 + 1	4 = 11 - 7
5 = 7 ...	5 = 10 ...	5 = 4 ...
5 = 7 - 2	5 = 10 - 5	5 = 4 + 1

The first term in the binary expression for difference is easily obtained as it is ordinary subtraction. Then the second term in the expression for difference is fixed so as to get the required difference. After handling a number of cases, students are excited to face the challenge of discovering the pattern in arriving at the second term of binary expression for the known difference. By handling the second terms of the minuend and the subtrahend, formulation of yet another oft-repeated rule 'change the sign and add' is made. It is also observed incidentally that each of the subtractions with second terms can be associated with its equivalent addition as displayed below:

- -		- +
6 + 3	is equivalent to	6 + 3
5 - 1		5 + 1
- -		- +
6 + 3	is equivalent to	6 + 3
2 + 2		2 - 2 and so on.

Multiplication is more challenging than addition and subtraction and yet it is not beyond the ability of upper primary school level students to tackle it and discover the familiar 'rule of signs' in multiplication.

$9 \times 7 = 63 \quad 9 = 6 + 3$

$7 = 5 + 2$

$63 = 5(6 + 3) + 2(6 + 3)$

$= 30 + 15 + 12 + 6$

$9 = 7 + 2$

$7 = 10 - 3$

$63 = 10(7 - 2) - 3(7 + 2)$

$= 70 + 20 - 21 - 6$

$9 = 11 - 2$

$7 = 4 + 3$

$63 = 4(11 - 2) + 3(11 - 2)$

$= 44 - 8 + 33 - 6$

$9 = 13 - 4$

$7 = 9 - 2$

$63 = 9(13 - 4) - 2(13 - 4)$

$= 117 - 36 - 26....$

$= 117 - 36 - 26 + 8$

Division does not lend itself to transitional management and hence consideration of it is postponed.

Structural Relations to Equations

Every addition fact yields not more than 2 associated subtraction facts and every subtraction fact in turn yields one addition fact and one associated subtraction fact.

$8 + 3 = 11 \quad - \quad 8 = 11 - 3 \quad \text{and} \quad 3 = 11 - 8$

$7 - 2 = 5 \quad - \quad 7 = 2 + 5 \quad \text{or} \quad 5 + 2$

$7 - 2 = 5 \quad - \quad 7 - 5 = 2$

These provide an effective background to handle simple equations involving + and - even before formal introduction to equations is made in higher class. Introduction of x or $\square$ could be made to replace any number in the above statements and the question be posed for finding its value.

The simplest is the straight forward one like

$8 + 3 = \square \quad \text{or} \quad 7 - 2 = \square$.

The rest involve transformation based on the relational statements.

$\square + 3 = 11 \rightarrow \square = 11 - 3$

$8 + \square = 11 \rightarrow \square = 11 - 8$

$7 - \square = 5 \rightarrow 7 - 5 = \square$

$\square - 2 = 5 \rightarrow \square = 2 + 5$

On similar lines every multiplication fact yields not more than two division facts and every division fact in turn yields one multiplication fact and one associated division fact

$5 \times 2 = 10 \rightarrow 10 \div 5 = 2$ and $10 \div 5 = 2$

$18 \div 6 = 3 \rightarrow 18 = 6 \times 3$ or 3×6

$18 \div 6 = 3 \rightarrow 18 \div 3 = 6$

These too provide an effective background to handle simple equations involving now multiplication and division.

$5 \times 2 = \boxed{} \rightarrow 5 \times 2 = 10$

$5 \times \boxed{} = 10 \rightarrow \boxed{} = 10 \div 5$

$\boxed{} \times 2 = 10 \rightarrow \boxed{} = 10 \div 2$

$18 \div \boxed{} = 3 \rightarrow 18 \div 3 = \boxed{}$

$\boxed{} \div 6 = 3 \rightarrow \boxed{} = 6 \times 3$

Notion of variable can be introduced at this juncture by means of sentences such as $x + y = 8$ and $xy = 12$, the number of solutions being related to the domain chosen for the variables.

Yet another context for the use of vairable and the corresponding letter symbolism presents itself whenever a formula is given in mensuration. For instance, consider rectangles of different dimensions given as positive integers and the areas
$4 \times 3 = 12$, $6 \times 4 = 24$, $8 \times 2 = 16$, $3 \times 5 = 15$
which on generalisation gives $l \times b = A$ or $e_1 \times e_2 = A$, l and b representing measures of length and breadth or e_1 and e_2 representing measures of edge 1 and edge 2 and A being the area of the rectangle. By extending the domain to include positive rational numbers, the same formula can be seen to hold good. Finally by extending the domain to include positive real numbers, the same formula gets assumed to be valid. Consideration of perimeters of rectangles and generalising the pattern in
$2(4 + 3) = 14$ $2(6 + 4) = 20$ $2(3 + 2) = 10$
$2(3 + 5) = 16$ $2(l + b) = P$ or $2(e_1 + e_2) = P$ is obtained.

Perimeters can also be introduced in the context of perimeters of regular polygons. If the perimeter $P = ka$ is taken as the formula for the periemeter of a regular polygon, **k** represents the parameter which changes with polygons of each kind. $k = 3$ gives the perimeter formula $P = 3a$ for the family of equilateral triangles; $k = 4$ gives the perimeter formula $P = 4a$ for the family of squares and so on.

Introduction of integers through postulation of opposites or additive inverses and relation of opposites to the additive identity zero, provides an excellent opportunity to solve certain problems with elegance. If it is asked what should be added to it would be easy to think +7 to get say -11, of the number that should be added to +7 to get zero and then add the required sum to fix the required addend. To solve

$$+7 + x = -11,$$

since $+7 -7 = 0$, $+7 -7 -11 = 0 - 11 \rightarrow +7 - 18 = -11$.

So the required addend is -18.

Similar situation obtains through the use of reciprocal or multiplicative inverse and multiplicative identity when positive rational numbers or fractional numbers for that matter, are introduced. To find the number by which 3/5 should be multiplied to get, say 8/13, it is easy to think of the number by which 3/5 should be multiplied to get 1 first and then multiply it by the required product to get the required multip-lier factor. To solve

$$3/5 \times X = 8/13,$$

since $3/5 \times 5/3 = 1$,

$$3/5 \times 5/3 \times 8/13 = 1 \times 8/13 \rightarrow 3/5 \times 40/39 = 3/13.$$

So the required multiplier is 40/39.

Computational Mathematics to Axiomatics

Abruptness in passing from computational and manipulative mathematics to propositional mathematics has been one of the major factors for resistance to learning in traditional curriculum and this situation was sought to be changed somewhat in the sixties when modernisation wave was witnessed. Introduction of

local axiomatics in lower classes facilitates learning mathematics not only as a body of computational skills and applicable tools but also as a quest for finality or truth in propositions.

Some questions that even children in primary classes can handle successfully with their reasoning powers are:

1) Can there be the last counting number? Since one more than the given counting number gives the next higher counting number, there cannot be the last counting number.

2) Can there be an even prime number other than 2? A number to be prime should have only two distinct factors. Assuming that there could be an even prime number other than 2, a contradiction would occur, as such an even prime would have 1, 2 and itself as factors, that is to say, three factors. So there cannot be more than one even prime number.

Some general propositions can also be considered and proved. Consider the classic proposition about any set of eight persons. At least two of any set of eight persons are born on the same day of the week. If it is a particular set and their days of birth are known, the truth of the proposition can be established by exhaustion or examination of all cases. If it is about any set, then the truth cannot be established by exhaustion. There is need for reasoning by contradiction. There are only seven days in a weak. If it is assumed that all the eight are born on different days, that would mean that a week should have 8 days. Since it would contradict the established fact, all the eight cannot be born on different days. If seven of them are born on different days, it would make seven days and the remaining one should be born on any of the seven days. So at least two of them should be born on the same day.

Early experience of disproving a statement through counter example will save learners from committing the oft-repeated mistake of offering verification as proof. The most appropriate place to

introduce disproof by counter example is when even and odd numbers, prime and composite numbers are introduced. A golden opportunity to make conjectures and examine their validity presents itself now. Some of the conjectures are

(1) a prime number + a prime number=a prime number 5+7=12. The counter example disproves it.

2) a prime number + a prime number = a composite number. The counter example 2 + 11 = 13 disproves it. So the sum of two prime numbers is neither prime nor composite. Similarly, that 1 is neither prime nor composite is also established, given the definition of a prime number as a number having only two distinct factors.

Elements of number theory provide also the opportunity to formulate conjectures and prove them. For instance, consider sets of three consecutive numbers : 1,2,3; 2,3,4; 3,4,5; etc. and their respective sums: 6,9,12, etc. The conjecture is that the sum of any three consecutive numbers is a multiple of 3. Assuming these consecutive natural numbers to be n-1, n,n+1, we find the sum is 3n and it is a multiple of 3. So the conjecture is proved and hence it becomes a theorem. Again by multiplying the end numbers of each triad of consecutive numbers and comparing the product with the square of the middle number, we get 1 x 3,4; 2 x 4, 9; 3 x 5, 16 etc. The conjecture is that the product of end numbers in a triad of three consecutive numbers is one less than the square of the middle number. Since $(n-1)(n+1) = n^2 - 1$, the conjecture is proved and it is found to be a theorem.

Some elementary geometrical propositítions can also be proved or disproved. A triangle should have at least two acute angles. This follows from the proposition that a triangle can at most have one right angle or one obtuse angle. To prove this, that the sum of the three angles of a triangle is two right angles is taken as a local axiom.

Conclusion

If these readiness inducing links could be well charted and incorporated in text books and teaching, school learners would become predisposed to have a more mature, meaningful and smooth passage from one thought process to another in mathematics.

CULTIVATION OF CLIMATE THROUGH MATHEMATICS CLUBS

3

Introduction

Mathematics clubs, as compared to science clubs, are still a novelty today in schools and in teacher-training colleges. The potential of mathematics clubs for motivating the learning of mathematics with greater enjoyment and less anxiety, for creating in learners a taste for mathematical thinking, for aiding the emergence of talent in the gifted, and for educating parents and people at large so as to enlist their support and patronage (Mmari, 1980), has yet to be tapped in full. Moreover, mathematics clubs can even help to prepare the climate for effecting changes in the school mathematics curriculum, in the methodology of teaching and in the programmes of evaluation.

Where they exist, mathematics clubs tend to flourish in post-secondary undergraduate institutions, where mathematics is offered as a major subject of study. Some secondary schools also have mathematics clubs, particularly those schools which have a few enthusiastic and devoted mathematics teachers on the staff. Mathematics clubs are not usually found in teacher-training colleges and in primary schools. With the fast-growing importance and influence of mathematics in the present-day world, there is an urgent need to encourage their growth in teacher-training institutions and in primary schools. This is particularly so in the developing countries of Africa, Asia, and Latin America. Where mathematics clubs in teacher-training colleges do exist, their influence upon students, teacher — educators and the community is considerable. Some students have even been inspired by them to organize, as part of their teaching practice, a pupils' mathematics club or to engage pupils in mathematical exposition. Through his involvement in a mathematics club, an interested and devoted teacher can even bridge the gap between the intended curriculum and implemented curriculum. He can thus expand the scope of examinations to include practical work in mathematics similar to the long-accepted practical work in science.

Organization

The organization of a mathematics club in a teacher-training institution should permit both ordinary membership drawn from students who take mathematics as part of their course work and the associate membership of students who do not.

Management of the affairs of the club should be entrusted to an elected body of officers drawn from the membership. This may comprise a president, a vice-president, a general secretary, a financial secretary, a treasurer, a social secretary and a publicity secretary. There should be a written constitution. This might make provision for co-opting other members with responsibilities, perhaps, for running a journal or a bulletin, maintaining a bulletin board, putting up displays in a showcase, etc. The posts of president, financial secretary and social secretary would normally be filled from the ranks of finalyear students. Other posts should go to more junior students, thereby ensuring stability and the continuity of the club as well as an equitable representation of members.

In primary school, the organization would normally be less elaborate. It might well be sufficient to elect or to nominate just three officers to run the club, such as a president, a secretary and a treasurer.

It would, of course, be prudent to assign to a mathematics club one or more members of staff to offer guidance on and supervision of programmes, and to ensure the proper management of funds in cases where funds are raised by membership fees or by donations from the student body.

As the name of a club is important in developing prestige and popularity, it is advisable to associate with it the name of some creative mathematicians such as Newton, Gauss, Euler, etc. This would remind students of the contributions great mathematicians have made to the growing edifice of mathematics.

Programmes and activities

A mathematics club should provide in its constitution for a well-designed annual programme of activities if it is to be influential in effecting attitudinal changes and evoking and nurturing the talents of gifted students. The programme may consist of periodical meetings, the issue of weekly bulletins, the organization of a mathematics week involving such activities as a students' symposium, a quiz, a 'make and take home models' workshop, learning activities and/or recreational mathematics expositions, mathematician masquerades to highlight the contributions of great mathematicians or to celebrate the birthdays of mathematicians, excursions, a mathematics fair, a mathematics education contest, some small-scale research and brain storming sessions for non-routine problem-solving. The objective should be to maximize the participation of members (*Srinivasan*, 1981).

There are also worthwhile and highly beneficial activities of a more ambitious type. For example, a number of institutions can agree to join together, to pool and share their resources so as to organize, on a co-operative basis , a screened, rolling relay mathematics exposition, contest, or fair open to the public. 'Screened' means making sure that the concepts in the curriculum are covered, but not repeated in the course of a number of years. 'Rolling relay' simply means that the exposition 'rolls' from one institution to another, while it 'relays' in the same institution. A certain number of consecutive week-ends is set apart to ensure that normal work is not disturbed, to reduce the strain of holding the exposition for a number of days in the same institution, to whet social expectancy, to raise the standards of the exposition through public and professional criticism and comment, and to provide scope for the participation of numerous students. Incidentally, such activity can generate interest in launching mathematics clubs, in other institutions. It can provide adult education. Another activity designed to satisfy the needs of mathematically inclined students who seek self-education is an 'exhibition course' in a topic outside the curriculum. Such a course, would, as the name implies, result in an exhibition of some kind. The course could also be rounded off with a test, an evaluation of responses and the award of prizes and certificates.

While examinations tend, in these days of mass education, to 'level down' standards of attainment, expositions can 'level up' standards of understanding through healthy emulation, challenge and concern.

Primary School

A mathematics club can enhance enormously the activities of a primary school. It is true that a primary-school teacher is not a specialist in mathematics. But the children's attitude towards mathematics is largely formed in their primary-school years. If children miss the excitement and enjoyment that should characterize their experience of learning mathematics, they easily develop negative attitudes towards mathematics, and are anxious to give it up at the very first opportunity that presents itself to them. A mathematics club can contribute a sense of confidence and help children to develop their mathematical gifts and tastes.

The difficulty is to find a primary-school teacher with enough confidence to organize a mathematics club for children. Yet, given administrative support and good guidance, a primary-school teacher can make a success of such an enterprise. And for most primary-school teachers, guidance and backing from above are essential to make up for an inadequate background and to compensate for any mathematical inadequecies there may be. Support can also be provided by the mathematics club in a teacher-training college. It can act as a resource centre for primary-school mathematics clubs in the vicinity; it can be the bank for lesson plans and for teaching aids and so cater to the needs of trained teachers and teachers under training.

Experience has shown that there are many activities that can appropriately be built into the programme of a primary-school mathematics club.

Here are just a few of them:

Making a collection of 'stories' that could account for a given numerical statement.

Making a collection of 'stories' that could give rise to a mathematical sentence; the sentence can be an equation or an inequality.

Number composition from a given lattice of points in some geometric shape such as a rectangle, a square, and combinations of such shapes.

Making mathematical statements by placing in alignment rectangular strips with measures indicated partially or fully on them.

Discovering patterns of numbers and patterns of shapes from formulae, or from extending a sequence.

Games based on 'think of a number'.

Creating algebraic expressions from language patterns.

Making easy generalizations and extensions.

Making appropriate definitions.

Reasoning from axioms and using a counter example to create a contradiction.

Building magic squares, magic triangles, magic crosses, magic hexagons and magic circles.

Using graphs as a model of behaviour, and interpreting graphs to forecast behaviour.

Abstraction from situations which have a common mathematical core of thought, but which seem to be dissimilar.

Demonstration talks on specific topics of the curriculum.

Mathematics without a chalk board.

Simple problem-solving sessions based on problems chosen for their interest, but suited to the background and ability of the children.

Mathematics in the environment-calender, clock, tiling, textile designs, braids, knots, seating, serializing, shopping, tailoring.

Howlers, fallacies and paradoxes drawn mostly from primary students' responses.

The mathematics club in a teacher-training college can help primary schools to introduce these activities through visits by student teachers. It can also organize, from time to time, inter-primary school

mathematics fairs or contests. Such contests can stimulate interest in and knowledge of 'new' techniques. For example, teaching aids are not yet thought to be indispensable to learning by mathematics teachers, and they are not used as widely as they should be. So by holding contests for primary-school teachers in techniques of improvisation in teaching mathematics, using waste materials like empty containers, paper, cardboard boxes, bottle tops, broom sticks, square-ruled sheets, etc. is encouraged. Contests for primary-school teachers can also be based on ingenuity in the treatment of a particular topic, and on the possibility of teaching a topic in the syllabus of a higher class to a lower class.

Bulletins can regularly be sent to schools, and feedback can help to improve the bulletin. A bulletin board can carry a quiz, with a 'discovery box' to collect pupils' responses. These can make it possible to detect and foster mathematical talent among primary-school pupils.

Caveat

A mathematics club should not be allowed to degenerate into an examination-coaching centre or special-classes centre. The tutor assigned to guide club actvities should do all he can to provide the members with opportunities to enjoy mathematics. The ethos of the club should be conducive to self-reliance and self-confidence, so as to facilitate the mathematical growth of members. Tutoring should be avoided. Members should be encouraged to feel free to commit mistakes and learn from them. The tendency to consider the club merely as an opening for distinguished people to be invited to give an occasional talk should be avoided. A club that does not plan for and secure the large-scale participation of its members can scarcely justify its existence.

Support

The inspectorates should be alive to the importance of mathematics clubs. They should maintain

a directory of them, and gather reports on their activities. When inspecting the schools and colleges of their zones or districts, they should make a point of recording the existence and work of associated mathematics clubs.

Each school or college can maintain a register of interested persons, drawn from professors, lecturers,engineers, scientists, doctors, etc. in the neighbourhood or in neighbouring institutions of higher learning.

Schools and colleges should also expect to be able to look to the national mathematics association in a country for guidance and support.

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SIGHTING THE VALUE OF Π

No secondary school mathematics program is complete without the formulae for the circumference and the area of a circle, which involve the introduction and the use of its approximate rational value 3 1/7 or 3.14.

As one who has gone about with almost religious sincerity in helping students develop mathematical ideas without being told, I have not been satisfied with the way I have been teaching π and the manner of its presentation in school textbooks. Over the years, I have been searching for a more natural and direct method for teaching π in such a way that the students see for themselves that 3 1/7 or 3.14 is the inevitable approximate value relating the circumference of a circle to its diameter and that π is an incommensurate number independent of scale.

The traditional approach in widespread use has been to (1) draw a circle of known diameter measured on a standard scale, (2) find the circumference of the circle with a piece of twine pressed carefully round the circle, (3) measure the length of the circumference by measuring the twine on the same scale, and (4) divide the circumference length by the diameter length to obtain an approximate value of π . The method is repeated for two or three more circles of different diameters, and, as in any determination of quantitative factor in a lab experiment, the mean of the various values of π is obtained. To get 3.14, the mean, as an approximate value for π becomes more often a matter of luck. The teacher has no choice but to enter the situation at this stage and announce that for all practical purposes, the value of π is to be taken as 3 1/7 or 3.14. Besides, a student does not find it easy to realize that π is independent of scale.

At long last, I hit upon a breakthrough in finding a more natural and innovative technique that would facilitate getting the value of π through the activities of students themselves. The use of scale as well as

the cumbersome, and hence the dull method of using thread and doing the division of C by d and finding the mean of numerous values of is discarded.

A new look at the Intercepts Theorem

The Intercepts Theorem comes in handy in achieving the breakthrough. The theorem states that *if three or more parallel lines make congruent intercepts on a transversal, they make congruent intercepts on any other transversal.* This geometric fact is verified by drawing equidistant parallel lines and transversals across them (fig. 1). The intercepts on any transversal are examined for congruence by the use of a pair of

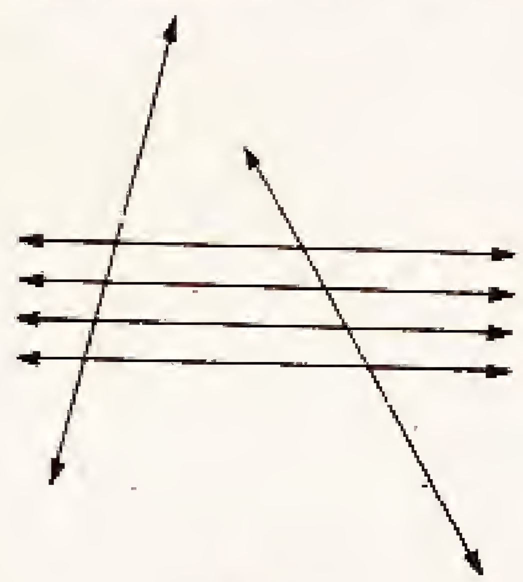

Fig. 1

dividers, and it is seen that the equidistant parallel lines make congruent intercepts on any transversal.

This fact is used in dividing a line segment into any finite number, say 4, of congruent segments (fig.2).

The end point of the segment to be multisected is chosen as the end point of a ray that does not contain the segment. With a pair of compasses, 4 congruent segments are marked off along the ray starting from the end point. Then the other end point of the line segment is joined to the last point of division on the ray. Lines parallel to this segment are drawn through the stepped-off points of division on the ray by using the pair of set squares or a set square and the ruler found in any box of geometrical instruments. The use of compasses and set squares secures the rigid location of a set of parallel equidistant lines that divide the line segment into the required number of congruent parts. I wondered if compasses and set squares could be discarded and instead a set of equidistant parallel lines are moved about and rotated to effect the congruent divisions of a line segment, and I discovered that it

could be done. The division of a line segment becomes at once simple and elegant.

What is needed is just a transparent ruled sheet of paper with equidistant parallel lines (TRS), the number of rulings being not less than ten. Figure 3 shows how a given line segment can be divided into two or more congruent parts by rotating the same TRS about the line segment. Incidentally, as the instant division of a line segment into the required number of congruent parts depends on the gap between the rulings and the length of the line segment, a suitable TRS can always be chosen.

Use in finding the relation (C, d) with respect to circles

The discovery of effecting instant division of a line segment into any finite number of congruent parts through rotation of the TRS provides an excellent application in sighting the value of π. Begin by constructing a ray (fig. 4).

A circular disc of cardboard, plastic, or metal can be used. A mark is made on its rim and a point taken on the ray. The disc is placed on the ray in such a manner that the mark on it coincides with the point on the ray. The disc is rolled along the ray without sliding till the mark coincides again with a point on the ray, indicating that the disc has rolled once, covering on the ray an interval taken to be equal to the circumference.

Using dividers with the legs spread out to the span of the diameter of the disc, the exploring student marks off the length of the diameter onto a segment and sees that the circumference of the disc is a little more than three times the diameter of the disc. The next step is to find out what part of the diameter is the little excess portion.

One way is to spread out the legs of the dividers to the span of the little excess portion and find out by stepping off to cover the diameter interval to the left, how many times the excess is contained in the diameter. It comes out to be almost seven times. The circumference is thus found to be almost 3 1/7 d. This procedure is repeated with discs of different

diamters and the general inductive result that the circumference of any circle is 3 1/7 times its diameter is obtained.

A more elegant and arresting method is to use a TRS. As before, a disc is taken; a mark is made on its rim; a ray is drawn showing up from a point on it four diameter-intervals; and the disc is rolled along the ray, starting with its mark in coincidence with the point on the ray. The spot where the mark comes back again on the ray is noted. It is easily seen that the circumference lies between 3d and 4d and is nearer 3d. A TRS is placed across the fourth diameter-interval, and by rotating it a student sights instantly that the little excess portion beyond the third diameter-interval is almost (1/7)d after finding that it is less than (1/2)d, (1/3)d, (1/4)d, (1/5)d, and (1/6)d (fig. 5). By rotating the TRS further, it is seen that the little excess portion is greater than (1/8)d, and so on. When the stage of ten parallel rulings across the fourth diameter-interval is viewed, one sights the circumference between 3.1d and 3.15d.

By using another TRS to divide the interval between 3.1d and 3.2d into ten equal parts, one sights that the circumference of the disc is almost 3.14d (fig. 6). When this experiment is repeated with discs of different diameters, one develops the readiness to admit that the circumference of a circle bears a constant relation to its diameter, and the constant is almost 3 1/7 or 3.14.

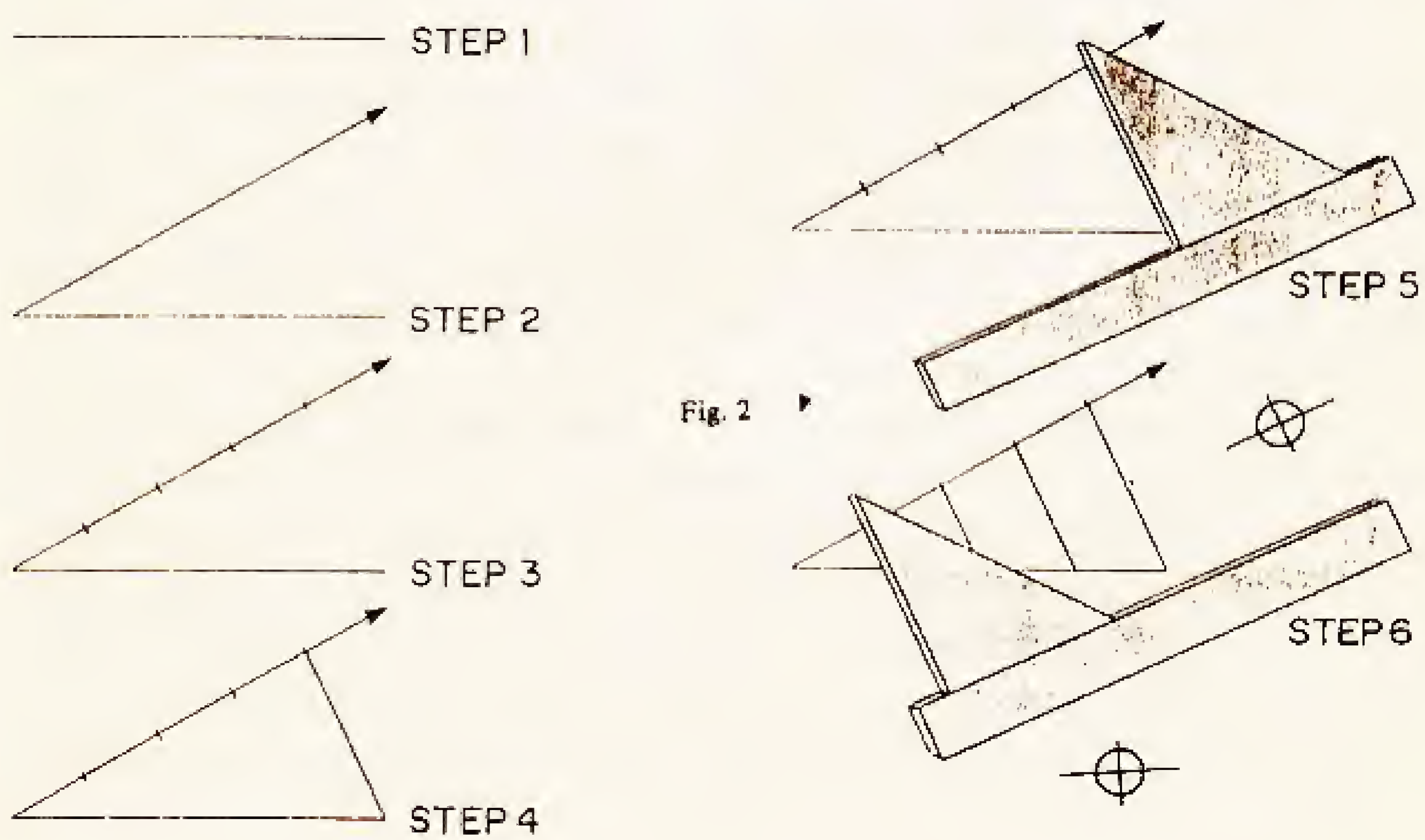

Fig. 2

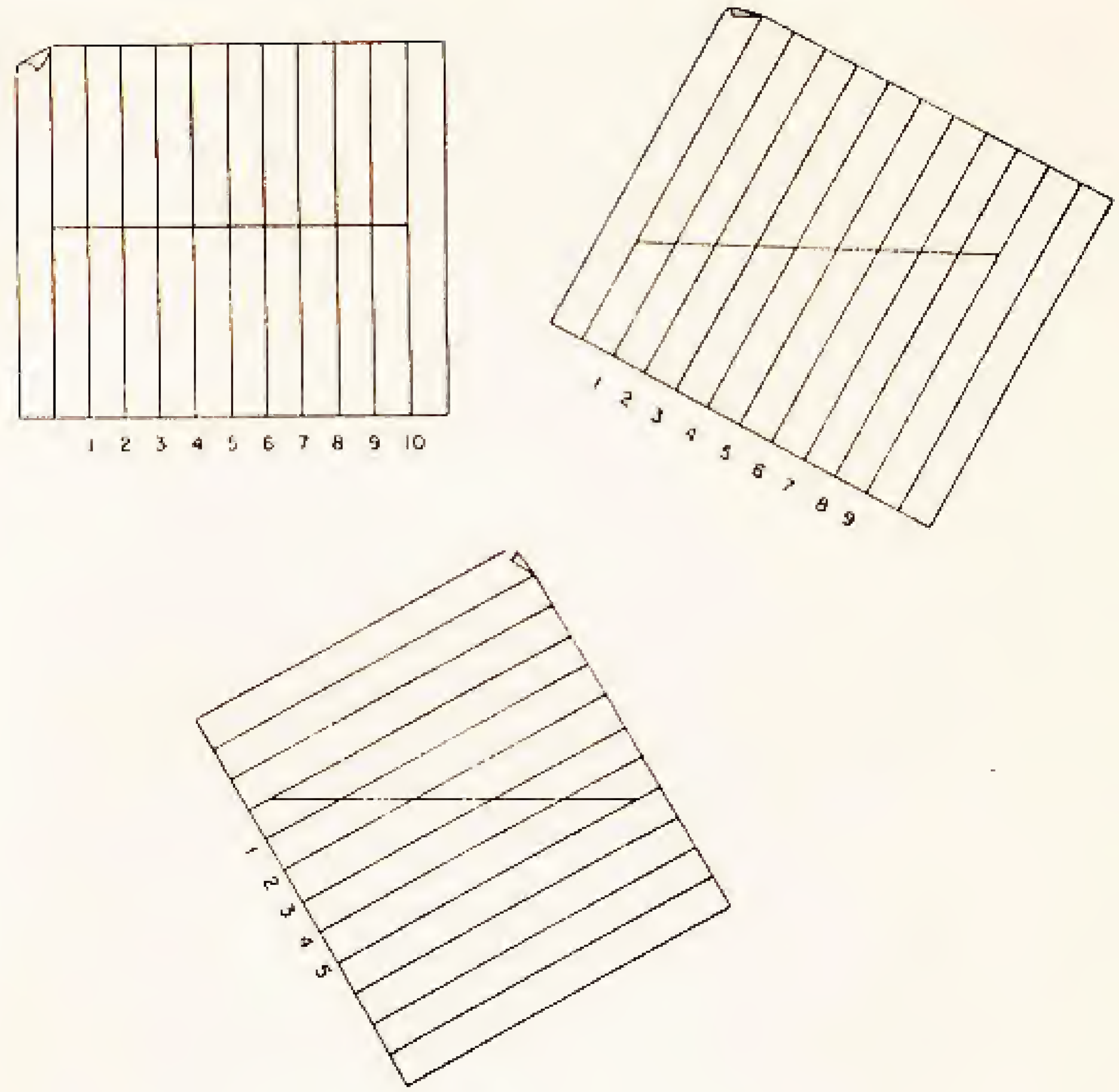

Fig. 3

Incidentally, a keen student is seen to observe, while rotating the TRS across the fourth diameter, that it is not possible to have the parallel rulings pass through the ends of the fourth diameter-interval and the point marking the end of the little excess portion. This gives students the readiness to appreciate that two line segments with no common measure do exist and hence admit the existence of incommensurate or irrational numbers when introduced.

A suggestion

This TRS technique is therefore natural and superior, hence all the more satisfying than the traditional one. With colored transparencies, the whole

drama of sighting the value of π can be presented to a large audience by means of an overhead projector.

The box of geometrical instruments that pupils use may be supplied with at least two transparent ruled sheets of convenient size, one with gaps between rulings different from those of the other and each having not less than ten rulings.

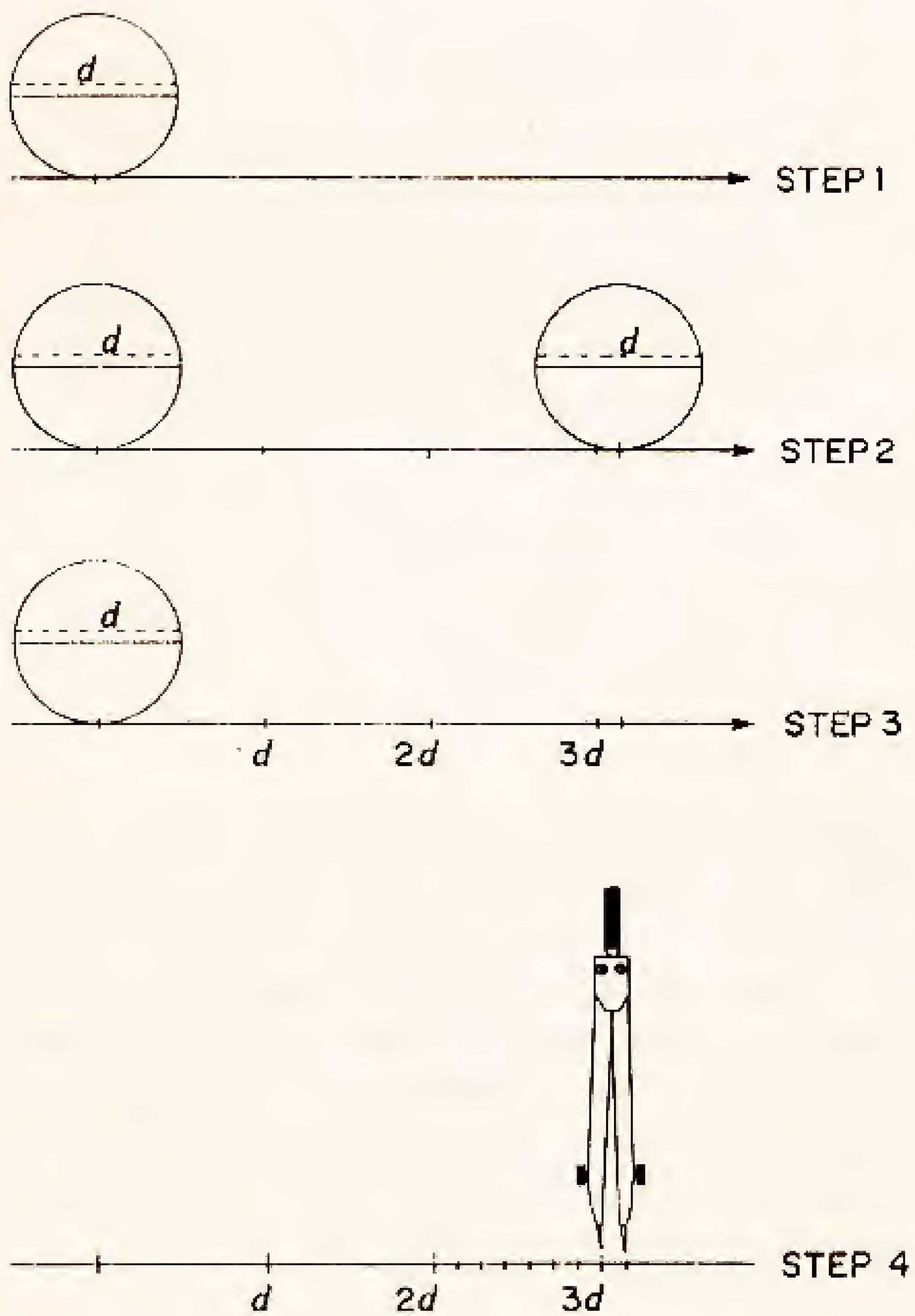

Fig. 4

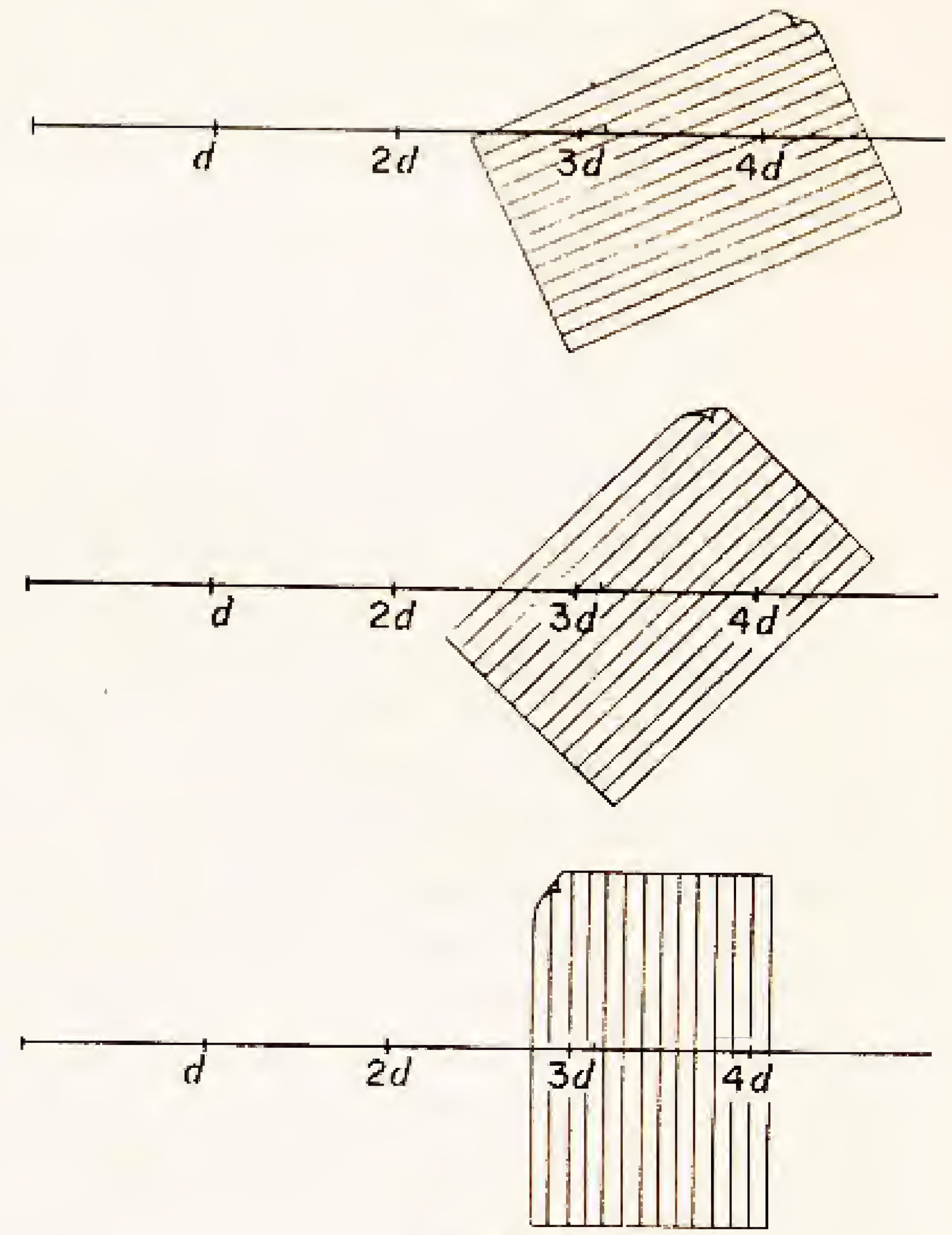

Fig. 5

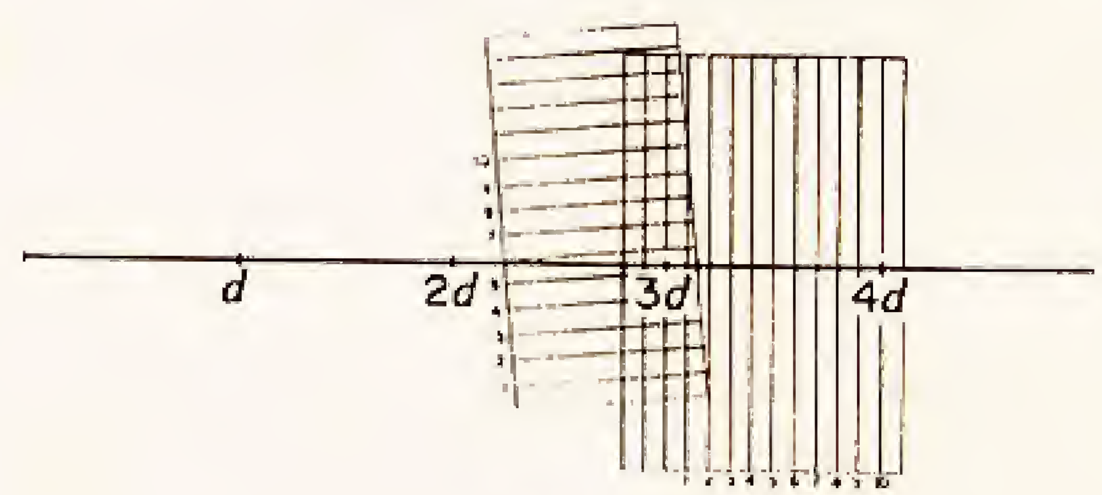

Fig. 6

DETECTION AND CARE OF THE GIFTED IN MATHEMATICS

WE ARE in the golden age of mathematics. But has the world been made golden for the gifted? All along they have been functioning, in spite of educational systems and regardless of handicaps and obstacles placed in their way through maltreatment born out of misunderstanding.

The year 1950 marked the first milestone in recognizing the special needs of this minority, whose development is of great significance to the strength and the future of a nation and of the world. That was the year when Dr. Guilford brought forth research findings in their favor. He spelled out the relation between IQ and creativity. It was clearly shown that high IQ often goes with low creativity. If the gifted are cold-shouldered, it is because of their creativity, their originality, their tendency not to conform, and their tendency to see something different and new even in widely accepted usages and procedures. They display reluctance to accept the obvious and to reach the final solution too soon. They have the ability to combine ideas that are usually considered unrelated.

It often requires a great deal of creativity on the part of teachers to recognize and appreciate creative contributions of the gifted. It is no wonder that teachers find them uncomfortable. Whereas the high IQ's are a source of pride and status to the teacher, the high CQ's (Creativity Quotients, if it can be put that way) are the cause of headache and agony for many a teacher.

Detection

The gifted cannot stand repetitions and reviews and routine methods of presentation. They get easily bored, as their pace of learning is high. They keep seeing new things and raising new questions - often baffling to the teacher. If a mathematics teacher entertains the notion that his her job is only to keep setting harder and harder problems that are to be given to keep the gifted busily engaged, it can be safely said that he has ceased to live mathematically. Mathematics, particularly in its modern developments, carries areas of simplicity, ingenuity, beauty and utility, like linear programming, finite mathematics, nonmetrical geometry, and theory of games, and the gifted can always be presented with rich fare to stir and exercise their creative imagination and thinking.

No teacher of mathematics could be effective if he were not fully familiar with the experiences of the gifted who have made a mark in the development of mathematics. Gauss, Euler, Abel, and Galois are examples.

Ramanujan

Here is presented in brief the romance of an Indian mathematical genius who rose from obscurity to fame within the short span of the thirty-two years he lived. He is Ramanujan (1887-1920), the prince among Indian mathematicians. The world knows him, not because of his teachers, who could only find him intractable, but because of Professor Hardy of England, who invited him and collaborated with him on equal terms in their common pursuit of mathematics.

Ramanujan read in Professor Hardy's tract *Orders of Infinity* that no formula existed for finding the number of primes less than a given number. He wrote to the professor a letter that started their historic association. Ramanujan was then only a matriculate, that is, a high school graduate with the record of failure in college. He had gone so far in expressing his genius through mathematics that he neglected other subjects, with the result that he became a dropout. Someone stood by him and helped him to get employment as a clerk in the office of the Port Trust, Madras, South India. Travel to foreign countries was not looked upon with favour in his days, and he had to overcome obstacles. Finally he went to England and by virtue of great scholarship rose to be accepted as the first Indian fellow of the Royal Society.

Even while Ramanujan was a student at Town High School, Kumbakonam, he had the research ability to adventure into uncharted territories. Students in the college sought his help in solving their problems. So they brought him advanced books and he found no

difficulty in mastering them. The one book that decidedly shaped his career in mathematics was Carr's Synopsis of Pure Mathematics.

Ramanujan left his notebooks as a priceless legacy to fellow minds of ages to come. Even as late as February 1965 the American Mathematical Monthly carried an article (by D H Lehmer) of further research on Ramanujan's contributions.

Teacher sympathetic to the gifted

Professor Hardy provides an example of a teacher sympathetic to the gifted. He had to help Ramanujan establish communication between his own world and the world of accepted scholarship. While acquainting him with different tools and techniques, Professor Hardy had to see that no damage was done to his spontaneity and creativity. He had also to cope with the phenomenon, quite common with the gifted, of highly creative individuality mixed with a great deal of ambiguity and absence of rigor in speculative flights.

It is great experience for teachers to see the notebooks and jottings by master minds in mathematics. Ramanujan's notebooks carry such interesting jottings, made during his school days. They reveal the sweep of imagination and penetrative insight he possessed even while at school.

One such jotting is worth quoting. To provide suspense, the problem is given first. Evaluate:

$$\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + 5\sqrt{\ldots}}}}}$$

Just think for a while whether it can be solved by high school mathematics!

Every ninth-grade teacher teaches

$$(n + 2)^2 = n^2 + 4n + 4$$

and $$n^2 + 4n + 3 = (n + 1)(n + 3).$$

It is exciting to see what magic and romance these have wrought in the mind of the gifted youngster. He takes

$$(n + 2)^2 = 1 + (n + 1)(n + 3),$$

and then writes

$$(n + 2) = \sqrt{1 + (n + 1)(n + 3)}.$$

Guided by an aesthetic feeling for form, he multiplies both the sides by n and arrives at the result

$$n(n+2) = n\sqrt{1 + (n+1)(n+3)}.$$

This is a beautiful property of four consecutive numbers represented by n, n+ 1, n + 2, and n + 3. Writing

$$n(n + 2) = f(n),$$

and getting

$$f(n + 1) = (n + 1)(n + 3),$$

he now finds himself in a fine take off stage. How could he stop?

$$f(n) = n\sqrt{1 + f(n + 1)}$$

$$= n\sqrt{1 + (n + 1)\sqrt{1 + f(n + 2)}}$$

$$= n\sqrt{1 + (n + 1)\sqrt{1 + (n + 2)\sqrt{1 + f(n + 3)}}}$$

Putting n = 1, he gets the wonderful result shown below.

$$1(1+2) = 1\sqrt{1 + (1+1)\sqrt{1 + (1+2)\sqrt{1 + (1+3)\sqrt{1 + \ldots}}}}$$

$$= 1\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + 5\sqrt{\ldots}}}}}$$

That is

$$3 = \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + 5\sqrt{1 + 6\sqrt{\ldots}}}}}}$$

Can a teacher envisage such a situation? All these steps are not given by him in his notebooks. He simply jotted down the result and proposed it as a problem for evaluation in the Journal of the Indian Mathematical Society. No wonder, only he could give the solution.

Ramanujan's notebooks bear testimony to his shifting interests. He starts with magic squares, involves himself in continued fractions, opens up the new territory of highly composite numbers that are ranked according to the increasing number of factors, enters the area of doubly periodic functions, conceives for the first time of fractional differentiation, develops divergent series, and sweeps on and on across different areas that hold his creative interest.

Care

Another important characteristic of the gifted is that of becoming bored with the textbooks. There is the classic example of Abel, who has gone on record with the statement that he read the masters, not the textbooks. The record of the truimphs, the break-throughs, and the pitfalls of the masters who have created the subject cast a spell which no mere text-book can provide. Student editions of books and notebooks of masters in mathematics should be brought out. The genius of Archimedes in working with a very inconvenient numeration system, the simple but profound contributions of Euler and Gauss in the theory of numbers, the absence of rigor in Newton, and the way Descartes expounded his coordinate geometry without the use of negatives - all provide a psychological appeal for the gifted youngsters. Textbooks rarely carry this appeal.

While answering questions put by the gifted, a teacher has to see that the mode of answers matches the maturity of the pupil. The special likes and dislikes shown by them while reacting to an approach should not be ignored. A book giving the treatment of school topics with different degrees of sophistication, different levels of approach, and various possibilities of extensions would be welcomed by every teacher of mathematics.

It is not to be taken that the mathematically gifted form a homogeneous class. Even among the gifted there can be seen roughly three levels. The ordinary level is characterized by a flair for generalization and extension. Higher than that is the ingenuity used to devise new techniques, like Farey's series, logarithms, and so on. The highest is that of breaking into a new line of thinking altogether, like Newton's calculus, Galois' group theory, and so on. At each level the gifted student needs special observation and care.

There cannot be a better motto for a teacher of the gifted than the Sanskrit saying, *"Sarvatrajayam-anvitchaet, Sishyadichaet paraajayam,"* which means, *"One should aspire for victory everywhere, but wish only for utter defeat by one's pupil."*

EDITOR'S NOTE. - *The foregoing is a condensed version of the talk given by Mr. Srinivasan as Fulbright Exchange Teacher of Mathematics on April 14, 1966, at the Annual Meeting of the NCTM at Americana Hotel, New York, USA.*

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